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LETTER TO THE EDITOR

Neutral kaon physics from the point of view of realism

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Abstract. We show that the quantum mechanical results for decaying, oscillating, and regenerating neutral kaons are compatible with treatments based on realism.

Neutral kaons provide an instructive example of a two-state system. A remarkable feature is the experimentally verified quantum mechanical prediction that, in the presence of weak interactions, neutral kaons will not only decay through the modes K_S and K_L (CP eigenstates) but will also display transitions ('oscillations') between the two states K^0 and $\bar{K}^0$ (strangeness eigenstates) [1]. In view of the realization we now have [2] that many quantum mechanical phenomena, such as the double-slit interference effect, can be reproduced by applying the point of view of realism (based on the notion that the quantum mechanical entities possess well defined attributes independent of measurements and are amenable to causal spacetime description), the question arises as to whether the quantum mechanical predictions for decaying and oscillating neutral kaons can also be reproduced using such a model. This issue is investigated in the present letter and we obtain an affirmative answer to the above question. We also analyse the phenomenon of coherent regeneration of K_S from the same point of view.

Such investigations are motivated by an outlook articulated in the following words of John Bell [3]: 'One wants to be able to take a realistic view of the world, to talk about the world as if it is really there, even when it is not being observed ... our business is to try to find out about it, and the technique for doing that is indeed to make models and to see how far we can go with them in accounting for the real world'. It is this spirit of realism that we pursue in this letter.

Based on 'realism' we assume that at any given time t an individual neutral kaon exists in a state which has simultaneously well defined eigenvalues of strangeness S and CP. Obviously, such a state has no meaning in quantum mechanics. We need then to introduce symbols for these non-quantum states and the following four possibilities are considered:

$$K_S(t),\ \bar{K}_S(t),\ K_L(t),\ \bar{K}_L(t)$$

where $K_S(t)$ denotes a kaon existing as K_S $(CP=+1)$ and K^0 $(S=+1)$, $\bar{K}_S(t)$ corresponds to a kaon as K_S and $\bar{K}^0$ $(S=-1)$, and similarly for $K_L(t)$ and $\bar{K}_L(t)$. We also introduce the probabilities

$$y_1(t),\ y_2(t),\ y_3(t),\ y_4(t)$$

corresponding to the states $K_S(t)$, $\bar{K}_S(t)$, $K_L(t)$ and $\bar{K}_L(t)$ respectively.

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Note that these probabilities are defined essentially in the sense of realism; the key assumption is that there exist elements of reality that determine the values of strangeness and of CP for every kaon, and the probabilities are taken to be relative frequencies of the different possibilities in the statistical ensemble.

To proceed further, we assume that the initial quantum mechanical state at $t=0$ is a pure K^0 state. At $t=0$ we, therefore, have

$$y_2(0)=y_4(0)=0 \qquad y_1(0)+y_3(0)=1. \tag{1}$$

Since it is experimentally clear that the two initial possibilities for K_S and K_L occur with equal frequencies we also have

$$y_1(0)=y_3(0)=\tfrac{1}{2}. \tag{2}$$

Experimentally verified quantum mechanical results for the initial K^0 state are given by

$$y_1(t)+y_3(t)=\tfrac{1}{4}[e^{-\gamma_S t}+e^{-\gamma_L t}+2\,e^{-\gamma t}\cos(\Delta m\cdot t)] \tag{3a}$$

$$y_2(t)+y_4(t)=\tfrac{1}{4}[e^{-\gamma_S t}+e^{-\gamma_L t}-2\,e^{-\gamma t}\cos(\Delta m\cdot t)] \tag{3b}$$

where $\gamma=(\gamma_S+\gamma_L)/2$, $\Delta m=m_S-m_L$, γ_S, γ_L are the decay widths of K_S, K_L, and m_S, m_L are masses of K_S, L_L respectively. Note that the (small) effects of CP violation will be ignored throughout our treatment. Interesting consequences of CP violation for the neutral kaons in the EPR context have been discussed by Datta *et al* [4].

We now assume that $y_1(t)$ and $y_3(t)$ are fractions of a common total given by (3a), reduced by factors $\frac{1}{2}e^{-\gamma_S t}$ and $\frac{1}{2}e^{-\gamma_L t}$ respectively. We can therefore write

$$y_1(t)=\tfrac{1}{2}e^{-\gamma_S t}y(t) \tag{4a}$$

$$y_3(t)=\tfrac{1}{2}e^{-\gamma_L t}y(t). \tag{4b}$$

By using (3a) and (4a, b) we get

$$y(t)=\tfrac{1}{2}[1+R(t)\cos(\Delta m\cdot t)] \tag{5}$$

where

$$R(t)=\frac{2\,e^{-\gamma t}}{e^{-\gamma_S t}+e^{-\gamma_L t}}=\frac{2\,e^{\gamma t}}{e^{\gamma_S t}+e^{\gamma_L t}}. \tag{6}$$

Consistent with (3a) we have then the following expressions for $y_1(t)$ and $y_3(t)$:

$$y_1(t)=\tfrac{1}{4}e^{-\gamma_S t}[1+R(t)\cos(\Delta m\cdot t)] \tag{7a}$$

$$y_3(t)=\tfrac{1}{4}e^{-\gamma_L t}[1+R(t)\cos(\Delta m\cdot t)]. \tag{7b}$$

Similarly, consistent with (3b) one gets the following expressions for $y_2(t)$ and $y_4(t)$:

$$y_2(t)=\tfrac{1}{4}e^{-\gamma_S t}[1-R(t)\cos(\Delta m\cdot t)] \tag{8a}$$

$$y_4(t)=\tfrac{1}{4}e^{-\gamma_L t}[1-R(t)\cos(\Delta m\cdot t)]. \tag{8b}$$

From now on we will limit ourselves to the case $CP=+1$, the generalization to $CP=-1$ being obvious. The rate equations for strangeness oscillations, including decays, are

$$y_1(t+dt)=y_1(t)[1-\{T_1+\gamma_S\}\,dt]+T_2y_2(t)\,dt \tag{9a}$$

$$y_2(t+dt)=y_2(t)[1-\{T_2+\gamma_S\}\,dt]+T_1y_1(t)\,dt \tag{9b}$$

where T_1 is the rate of transition from $S=+1$ to $S=-1$ and T_2 is the opposite rate from $S=-1$ to $S=+1$. In writing (9a) and (9b) we have assumed that CP eigenvalue is a fixed property of every kaon (there is no possibility of transitions between (K_S, $\bar{K}_S$) and (K_L, $\bar{K}_L$) states), given the quantum mechanical result

$$\langle K_S(t)|K_L(0)\rangle = \langle K_L(t)|K_S(0)\rangle = 0. \tag{10}$$

From (9a) and (9b) we have the differential equations

$$y_1' = \frac{\mathrm{d}y_1}{\mathrm{d}t} = -(T_1+\gamma_S)y_1 + T_2 y_2 \tag{11a}$$

$$y_2' = \frac{\mathrm{d}y_2}{\mathrm{d}t} = -(T_2+\gamma_S)y_2 + T_1 y_1. \tag{11b}$$

Then the equation of compatibility between quantum mechanics and realism in the context of neutral kaon physics boils down to the following one: consistent with the expressions for $y_1(t)$ and $y_2(t)$ given by (7a) and (8a) respectively, can one have T_1 and T_2 which satisfy the equations (11a) and (11b) and are finite as well as positive?

To examine this question we proceed as follows. Writing

$$y_1 = \mathrm{e}^{-\gamma_S t} W_1 \qquad y_2 = \mathrm{e}^{-\gamma_S t} W_2 \tag{12}$$

equations (11a) and (11b) become respectively

$$W_1' = -T_1 W_1 + T_2 W_2 \tag{13a}$$

$$W_2' = -T_2 W_2 + T_1 W_1 \tag{13b}$$

whence using (7a, b) and (12), (13a, b) both become

$$R'(t)\cos(\Delta m\cdot t) - R(t)\Delta m \sin(\Delta m\cdot t) = 4(T_2 W_2 - T_1 W_1). \tag{14}$$

Notice that (14) can be written as

$$R(t)\Delta m \sin(\Delta m\cdot t) - R'(t)\cos(\Delta m\cdot t) = B + AR(t)\cos(\Delta m\cdot t) \tag{15}$$

where

$$T_1 + T_2 = A \qquad T_1 - T_2 = B. \tag{16}$$

From (15) and (16) one obtains

$$T_1 = \tfrac{1}{2}\{A[1-R\cos(\Delta m\cdot t)] + R\Delta m\sin(\Delta m\cdot t) - R'\cos(\Delta m\cdot t)\} \tag{17a}$$

$$T_2 = \tfrac{1}{2}\{A[1-R\cos(\Delta m\cdot t)] - R\Delta m\sin(\Delta m\cdot t) + R'\cos(\Delta m\cdot t)\}. \tag{17b}$$

Our basic question is now reduced to the following one: can one choose A in such a way that T_1 and T_2 are finite and positive? The answer is 'yes' and the most simple ansatz is given by

$$A = \Delta m = m_S - m_L. \tag{18}$$

With the choice (18) for A we have calculated numerically T_1 and T_2 for a wide range of values for $\Delta m\cdot t$. The curves for $T_1/\Delta m$ and $T_2/\Delta m$ as functions of t are shown in figure 1. Note that T_1 and T_2 are functions of time. It needs to be stressed that, in principle, time-dependent transition rates have no non-physicality associated with them. There are well known instances of transition rates varying with time, such as the atomic transition rates in the presence of time-varying external fields. It is also possible to construct 'internal' models of the kaons, based on realism, that give rise to time-dependent rates for strangeness jumps. It is clear that T_1 and T_2 remain finite

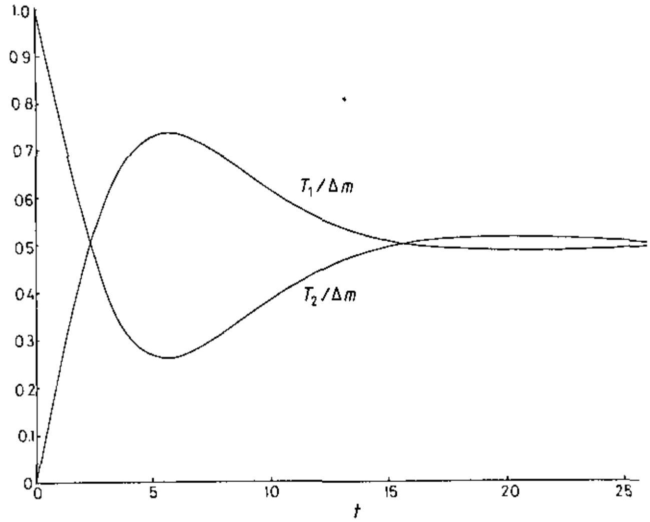

Figure 1. The dimensionless rates $T_1(t)/\Delta m$ and $T_2(t)/\Delta m$ as functions of t. Time unit is $\frac{1}{16}\pi\Delta m$.

and positive for the choice (18). As one can see from figure 1, $T_1/\Delta m$ and $T_2/\Delta m$ both go to the value 1/2 when $t\to\infty$: they are already very near to 1/2 for all $t \geqslant \pi/\Delta m$. Below $t=\pi/\Delta m$, the two curves are well above zero for all values of t except for the region just above $t=0$ in the case of $T_1/\Delta m$. We have carefully checked, both numerically and with a power expansion, that T_1 is always non-negative in this region. In fact, it is easy to see that for low values of t one can write the linear approximation

$$\frac{T_1(t)}{\Delta m} \simeq \Delta m \cdot t\left(1+\frac{\gamma_S^2}{4\Delta m^2}\right) \geqslant 0 \tag{19a}$$

$$\frac{T_2(t)}{\Delta m} \simeq 1-\Delta m \cdot t\left(1+\frac{\gamma_S^2}{4\Delta m^2}\right) > 0. \tag{19b}$$

Table 1 gives the results of numerical calculations of $T_1/\Delta m$ and $T_2/\Delta m$, as given by (17*a*, *b*) with the choice (18) for *A*. It shows that the linear approximation is good

Table 1

$16\cdot\Delta m\cdot t/\pi$	$T_1/\Delta m$	$T_2/\Delta m$
0.0	0.000	1.000
0.1	0.021	0.979
0.2	0.042	0.958
0.3	0.064	0.936
0.4	0.085	0.915
0.5	0.107	0.893
0.6	0.129	0.871
0.7	0.151	0.849
0.8	0.173	0.827
0.9	0.196	0.804
1.0	0.218	0.782

at least up to $t=\pi/16\Delta m$. Therefore, there can be no question of $T_1(t)$ becoming negative for low values of t. This completes the demonstration of compatibility between quantum mechanics and realist description of neutral kaons which decay and oscillate between K^0 and $\bar{K}^0$ states.

Let us now recapitulate the essence of the regeneration phenomenon. If one has initially a pure state K^0 beam and allows it to travel in vacuum for a time sufficiently longer than the K_S lifetime ($\simeq 10^{-10}$ s), one is left with a pure K_L beam, where in quantum theory

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle). \tag{20}$$

If now the K_L beam is made to interact with a slab of material, the $|\bar{K}^0\rangle$ and $|K^0\rangle$ components are absorbed with different cross sections in the material. While $\bar{K}^0$ strongly interacts with the nucleons to give Λ and Σ particles, K^0 is predominantly elastically scattered. The state $|\Psi\rangle$ of the beam emerging from the slab is then given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(f|K^0\rangle + \bar{f}|\bar{K}^0\rangle) = \tfrac{1}{2}(f+\bar{f})|K_L\rangle + \tfrac{1}{2}(f-\bar{f})|K_S\rangle[\bar{f} < f \leq 1]. \tag{21}$$

Since $f \neq \bar{f}$, the $|K_S\rangle$ component has been regenerated from $|K_L\rangle$. If the slab is thick enough, the entire $|\bar{K}^0\rangle$ component interact with nucleons and get converted into Λ and Σ particles. Then the component that emerges from the slab is essentially $|K^0\rangle$ which has both $|K_L\rangle$ and $|K_S\rangle$ components. From the point of view of realism, the incident beam had particles in a state corresponding to the state vector $|K_L\rangle$, i.e. the particles could decay only through the K_L mode. The particles that emerge from the thick slab are essentially the ones which have been transmitted without any exchange of momentum or energy. Nevertheless, after emerging from the slab, these particles are in a state corresponding to the state vector

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_L\rangle + |K_S\rangle) \tag{22}$$

which means that these particles can now decay through both the K_L and K_S modes. Within the framework of realism this is a striking example of the action of the physically real 'empty waves' (devoid of momentum and energy), corresponding to the total wavefunction (22), on the emergent particles affecting their innate characteristics such as the decay modes and strangeness quantum number. In other words, the kaonic particle possessing energy and momentum and capable of decaying, e.g. into pions, has physical properties depending on the nature of the empty wave that surrounds it. When that wave gets modified in the slab according to

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \rightarrow |K^0\rangle$$

the particles acquire a new property, namely the possibility of decaying in the K_S mode. Following the Einstein–de Broglie approach, this is a striking example of an interplay between the wave and particle aspects of the kaons. This picture, strongly suggested in the case of coherent regeneration, also implies that the elements of reality dealt with in the case of decaying and oscillating kaons must result from the interplay between the wave and particle aspects of the kaons as well.

The particular form of realism assumed in this letter is the one that follows from the application of the EPR reality criterion, in conjunction with the locality condition, to the EPR-type example involving separated and correlated K^0-$\bar{K}^0$ pairs [5]. It is remarkable that exactly the same elements of reality which must necessarily be introduced for the correlated K^0-$\bar{K}^0$ pairs reproduce exactly the physics of single kaons. The remaining issue is, therefore, the compatibility of locality with the quantum mechanical predictions for the correlated K^0-$\bar{K}^0$ pairs. On this point one may refer to the recent paper by Six [6] who has claimed to have reproduced the quantum mechanical results for the K^0-$\bar{K}^0$ system with a specific local realistic model. However, his model contains negative probability functions which are physically unacceptable. A detailed criticism of Six's will be given elsewhere [7].

Having demonstrated the compatibility between quantum mechanics and realism in the context of decaying, oscillating, and regenerating neutral kaons, the next step will be to investigate this issue with reference to the correlated K^0-$\bar{K}^0$ pairs. Such investigations assume considerable importance, particularly in view of the proposed Φ factory project at Frascati [8] which would enable crucial measurements of joint probabilities for the K^0-$\bar{K}^0$ system.

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